$U_A(1)$ symmetry breaking, scalar mesons and the nucleon spin problem in an effective chiral field theory

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(February 1, 2008)

Abstract

We establish a relationship between the scalar meson spectrum and the $U_A(1)$ symmetry-breaking 't Hooft interaction on one hand and the constituent quark's flavor-singlet axial coupling constant $g_{A,Q}^{(0)}$ on the other, using an effective chiral quark field theory. This analysis leads to the new sum rule $g_{A,Q}^{(0)} \left[m_{\eta'}^2 + m_\eta^2 - 2m_K^2 \right] \simeq -m_{f_0'}^2 - m_{f_0}^2 + 2m_{K_0^*}^2$, where η', η, K are the observed pseudoscalar mesons, K_0^* is the strange scalar meson at 1430 MeV and f_0, f_0' are "the eighth and the ninth" scalar mesons. We discuss the relationship between the constituent quark flavor-singlet axial coupling constant $g_{A,Q}^{(0)}$ and the nucleon one $g_{A,N}^{(0)}$ ("nucleon's spin content") in this effective field theory. We also relate $g_{A,Q}^{(0)}$, as well as the flavour-octet constituent quark axial coupling constant $g_{A,Q}^{(1)}$ to vector and axial-vector meson masses in general as well as in the tight-binding limit.

PACS numbers: 11.30.Rd, 11.40.Ha, 12.40.Yx

Typeset using REVTEX

I. INTRODUCTION

A few years ago the present author and simultaneously and independently a group at the University of Bonn [1,2] suggested that the $U_A(1)$ symmetry breaking 't Hooft interaction, induced by instantons in QCD, is a clue to the problem of scalar meson classification (see p. in Ref. [3]) The gist of that calculation is embodied in the sum rule $m_{\eta'}^2 + m_\eta^2 - 2m_K^2 \simeq -m_{f_0'}^2 - m_{f_0}^2 + 2m_{K_0^*}^2$, where η', η, K are the observed pseudoscalar mesons, K_0^* is the strange scalar meson at 1430 MeV and f_0, f_0' are "the eighth and the ninth" scalar mesons. This relation enables one to determine the mass of the second f_0 meson as a function of the first one and the (strange) scalar kaon mass (the r.h.s. are the p.s. masses which are known).

Although one specific pair $(f_0(980), f_0'(1500))$ of 0^+ states was proposed as leading candidates by both sets of investigators [1,2], it was not long before Eq. (1) was used to: (1) select other candidate f_0, f_0' pairs [4,5]; (2) try to identify the lowest-lying 0^+ glueball [5]; (3) look for radially excited 0^+ states [6]; etc. We note, however, that Eq. (1) was derived in the simplest possible effective chiral model with the 't Hooft interaction [7].

In this note I report the results of an analogous calculation in an extended effective chiral model with the 't Hooft interaction. This model includes dynamically bound ("composite") vector- and, due to chiral symmetry also axial-vector meson states. The mixing of the pseudoscalar (0^-) and the longitudinal component of the axial-vector (1^+) states leads to an important phenomenon: the reduction of the constituent quark axial coupling constant. This reduction, however, occurs at different levels in the flavour singlet- and octet channels. We derive a modified mass relation

$$g_A^{(0)} \left[m_{\eta'}^2 + m_\eta^2 - 2m_K^2 \right] \simeq -m_{f_0'}^2 - m_{f_0}^2 + 2m_{K_0^*}^2, \tag{1}$$

in our effective field theory that is a consequence of the broken flavor $SU(3)$ and $U_A(1)$ symmetries. Here $\eta', \eta, K, K_0^*, f_0, f_0'$ are as above and $g_{A,Q}^{(0)}$ is the constituent quark's flavor-singlet axial coupling constant. The nature of this symmetry breaking is also identical to the case without axial-vector states, only the size of the effect on scalar mesons is now reduced by a factor of $g_A^{(0)}$.

It is well known that the flavour-singlet nucleon axial coupling constant is: (a) much smaller than the flavour-octet one; and (b) measured in deep-inelastic lepton scattering where it leads to the so-called "nucleon spin problem", which, in turn is related to the $U_A(1)$ problem [8]. Manifestly, the flavour-singlet channel longitudinal-axial-vector–pseudoscalar mixing depends on the $U_A(1)$ symmetry-breaking 't Hooft interaction and hence on the $U_A(1)$ symmetry-breaking 't Hooft mass that enters the l.h.s. of the sum rule (1).

II. THE MODEL AND THE METHODS

A. Definition of the effective chiral field theory

Our effective chiral field theory ought to have the same chiral symmetries as QCD, yet it ought to be more tractable than QCD itself. So we construct a chiral Lagrangian with the requisite symmetries solely out of quark fields. The $U_L(N_f) \times U_R(N_f)$ symmetric part of the QCD Lagrangian (quark-gluon interactions) is replaced by a set of four four-fermion

interactions (one scalar + pseudoscalar and three vector and/or axial-vector) with the same symmetry, whereas the $U_A(1)$ symmetry-breaking term is given by the 't Hooft interaction generated by instantons in QCD [7]

$$\mathcal{L}_{\mathrm{N_f}=3} = \bar{\psi}[i\slashed{\partial} - m^0]\psi + G\sum_{i=0}^{8} [(\bar{\psi}\boldsymbol{\lambda}_i\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\lambda}_i\psi)^2]$$
$$- K\left[\det\left(\bar{\psi}(1+\gamma_5)\psi\right) + \det\left(\bar{\psi}(1-\gamma_5)\psi\right)\right]$$
$$- G_V^{(1)}\sum_{i=1}^{8}\left[(\bar{\psi}\gamma_\mu\boldsymbol{\lambda}_i\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\lambda}_i\psi)^2\right]$$
$$- \left[G_V^{(0)}(\bar{\psi}\gamma_\mu\boldsymbol{\lambda}_0\psi)^2 + G_A^{(0)}(\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\lambda}_0\psi)^2\right] , \tag{2}$$

The latter interaction is a flavour space-determinant $2N_f$-quark interaction, where N_f is the number of flavors. In the flavour-octet channels one must have $G_V^{(1)} = G_A^{(1)}$ in order to preserve the chiral symmetry, whereas in the flavour-singlet channel the two coupling constants need not coincide $G_V^{(0)} \neq G_A^{(0)}$.

Four vector and three axial-vector (isovector) currents are conserved in the chiral limit and the pion is massless. The chirally symmetric field theory described by $\mathcal{L}_{\mathrm{N_f}=3}$ in both its original ($G_{V,A} = 0$) and extended versions ($G_{V,A} \neq 0$) exhibits spontaneous symmetry breakdown into a nontrivial ground state with constituent quark mass generation and a finite quark condensate, when dealt with non-perturbatively. It is therefore essential to have a non-perturbative approximate solution that preserves the underlying chiral symmetry. To leading order in $1/N_C$ such an approximation is described by two Schwinger-Dyson [SD] equations: the gap equation and the Bethe-Salpeter [BS] equation. [A chirally symmetric approximation going beyond the leading order in $1/N_C$ has been developed in Ref. [9].] This approximation involves (self-consistent) one-loop diagrams that are infinite without regularization, so one must introduce some sort of regularization at this level of approximation, say a momentum cut-off Λ. This cutoff has a natural explanation in terms of instanton size within 't Hooft's QCD derivation [7] and ought to be around 1 GeV, where indeed it has traditionally been in the NJL model. The Lagrangian (2) defines a perturbatively (Dyson) non-renormalizable field theory, so we keep the cutoff which sets the 4-momentum scale in the theory.

B. One-body (gap) equation

We start with the gap equations that determines the quark self-energies, i.e., masses. To leading order in $1/N_C$ they read:

$$m_u = m_u^0 - 4G\langle\bar{u}u\rangle + 2K\langle\bar{d}d\rangle\langle\bar{s}s\rangle \tag{3a}$$
$$m_d = m_d^0 - 4G\langle\bar{d}d\rangle + 2K\langle\bar{u}u\rangle\langle\bar{s}s\rangle \tag{3b}$$
$$m_s = m_s^0 - 4G\langle\bar{s}s\rangle + 2K\langle\bar{d}d\rangle\langle\bar{u}u\rangle \tag{3c}$$

depicted in Fig. 1b. The quark condensates

$$\langle\bar{q}q\rangle = -iN_C\mathrm{tr}S_F(x,x)^q = -4iN_C\int\frac{d^4p}{(2\pi)^4}\frac{m_q}{p^2 - m_q^2 + i\varepsilon}, \quad q = u, d, s \ ; \tag{4a}$$

$$\langle\bar{\psi}\psi\rangle = \langle\bar{u}u\rangle + \langle\bar{d}d\rangle + \langle\bar{s}s\rangle = -iN_C\mathrm{tr}S_F(x,x) \quad . \tag{4b}$$

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are to leading approximation all equal. This allows the following generic form of the (formerly coupled) gap equations

$$m_q = m_q^0 + 16iN_C(G + \frac{1}{2}G_{tH})\int \frac{d^4p}{(2\pi)^4}\frac{m}{p^2 - m^2}$$

$$= m_q^0 - \frac{4}{3}G_{eff}\langle\bar{\psi}(x)\psi(x)\rangle_0 \quad , \tag{5}$$

that can be regulated either by introducing a Euclidean cut-off Λ_E, or by following Pauli and Villars (PV) [10]. Here $G_{tH} = -K\langle\bar{q}q\rangle$.

The gap equation establishes a relation between the constituent quark mass m and the two free parameters $G_{eff} = G + \frac{1}{2}G_{tH}$ and Λ. This relation is not one-to-one, however: there is a (double) continuum of allowed G_{eff} and Λ values that yield the same nontrivial solution m to the gap equation. Even if we knew the precise value of m, which we don't, there is still a great deal of freedom left in the (G_{eff}, Λ) parameter space. Blin, Hiller and Schaden [11] showed how to eliminate one of these two continuum degeneracies in the $G_A^{(1)} = 0$ version of the theory by fixing the value of the pion decay constant $f_p = f_\pi(G_A^{(1)} = 0)$ at the observed value 93 MeV. This procedure was extended to the $G_A^{(1)} \neq 0$ case in Ref. [12]. We shall follow the latter procedure in this paper. Perhaps the most important consequence of the fixed f_π constraint is the finite renormalization: (i) of the "bare" ($G_A^{(1)} = 0$) pion decay constant f_p to f_π; (ii) of the bare pseudoscalar πqq coupling g_p to g_π, and (iii) of the constituent quark axial coupling from unity to $g_A \leq 1$ according to

$$\left(\frac{g_p}{g_\pi}\right)^2 = \left(\frac{f_\pi}{f_p}\right)^2 = g_A^{(1)} = 1 - 8G_A^{(1)}f_\pi^2 \ . \tag{6}$$

Our f_π-fixing procedure yields a separate m vs. Λ curve for every value of $g_A^{(1)}$, see Fig. 2. Similarly we have in the flavour-singlet channel

$$\left(\frac{g_p}{g_{\eta_0}}\right)^2 = \left(\frac{f_{\eta_0}}{f_p}\right)^2 = g_A^{(0)} = 1 - 8G_A^{(0)}f_\pi^2 \ . \tag{7}$$

Moreover, this finite renormalization is the source of significant changes in the vector- and axial-vector mesons' spectra in this effective field theory as discussed in Ref. [12].

C. Two-body (Bethe-Salpeter) equation

The second Schwinger-Dyson equation is an inhomogeneous Bethe-Salpeter (BS) equation

$$-\mathbf{D} = 2\mathbf{G} - 2\mathbf{G}\,\mathbf{\Pi}\,\mathbf{D}, \tag{8}$$

describing the scattering of quarks and antiquarks Fig. 1(b). Here $\mathbf{G}$ is the (effective) coupling constant matrix for the relevant channels and $\mathbf{\Pi}$ is the relevant polarization function matrix. The quartic fermion interaction allows one to schematically write the generic solution for the BS propagator as the geometric sum depicted in Fig. 1(b). The form of the interaction in Eq. (2) gives rise to scattering in 36 ($= 9 \times 4$) flavour/spin-parity channels.

Solutions to the BS equation in the scalar/pseudoscalar channels As explained in Ref. [1], working directly with the sixth-order operator Eq. (2) in the BS equation is, for this purpose, equivalent to using the *quartic* "effective mean-field 't Hooft self-interaction Lagrangian", as constructed in that reference,

$$\mathcal{L}_{\text{eff}}^{(4)} = \bar{\psi}[i\slashed{\partial} - m^0]\psi + \left[K_0^{(-)}(\bar{\psi}\boldsymbol{\lambda}_0\psi)^2 + \sum_{i=1}^{8} K_i^{(+)}(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_i\psi)^2\right]$$
$$+ \left[K_0^{(+)}(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_0\psi)^2 + \sum_{i=1}^{8} K_i^{(-)}(\bar{\psi}\boldsymbol{\lambda}_i\psi)^2\right]$$
$$+ G_V^{(1)} \sum_{i=1}^{8} \left[(\bar{\psi}\gamma_\mu\boldsymbol{\lambda}_i\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\lambda}_i\psi)^2\right]$$
$$+ \left[G_V^{(0)}(\bar{\psi}\gamma_\mu\boldsymbol{\lambda}_0\psi)^2 + G_A^{(0)}(\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\lambda}_0\psi)^2\right] \quad, \tag{9}$$

where

$$K_0^{(\pm)} = G \pm K\langle\bar{q}q\rangle = (G \mp G_{\text{tH}}) \tag{10a}$$
$$K_i^{(\pm)} = G \mp \frac{1}{2}K\langle\bar{q}q\rangle = \left(G \pm \frac{1}{2}G_{\text{tH}}\right) , \quad i = 1,\dots,8 . \tag{10b}$$

The gap equation (5) can now be written as

$$m_q = m_q^0 - 4K_1^{(+)}\langle\bar{q}q\rangle \quad, \tag{11}$$

i.e., we have $K_1^{(+)} = G_{\text{eff}}$. This fact ensures that there are eight Nambu-Goldstone bosons in the chiral limit $m_q^0 \to 0$.

Because there is mixing between the pseudoscalar and the longitudinal axial vector channels, all the objects in the BS equation are 2×2 matrices. The solutions to the BS matrix Eq. (8) for the propagator matrix $\mathbf{D}$ become

$$-\mathbf{D} = 2\mathbf{G}(1 - 2\boldsymbol{\Pi}\,\mathbf{G})^{-1} = (1 - 2\mathbf{G}\,\boldsymbol{\Pi})^{-1}(2\mathbf{G}) \tag{12}$$

The meson masses are read off from the poles of the relevant propagators, which in turn are constrained by the gap Eq. (5). Upon introducing explicit chiral symmetry breaking in the form of non-zero current quark masses m_i^0, one finds in the octet channel

$$\mathbf{D}_{ab}^{(8)}(q^2) = \frac{1}{\left(q^2 F(q^2) - m_{ab}^2(1 + 8G_A^{(1)}f_a^p f_b^p F(q^2))\right)}$$
$$\times \begin{pmatrix} g_p^2\left(1 + 8G_A^{(1)}f_p^a f_p^b F(q^2)\right) & 4G_A^{(1)}m\sqrt{q^2}F(q^2) \\ 4G_A^{(1)}m\sqrt{q^2}F(q^2) & 2G_A^{(1)}q^2 F(q^2) \end{pmatrix} . \tag{13}$$

Here m_{ab}^2 is due to the current quark mass matrix m_0, e.g. the diagonal mass-squared matrix elements $f_a m_{\text{qmass}}^2 f_b = f_a m_{ab}^2 f_b = \delta_{ab} m_a^2 f_a^2$ are

$$m_\pi^2 f_\pi^2 = -\left[m_u^0\langle\bar{u}u\rangle_0 + m_d^0\langle\bar{d}d\rangle_0\right]; \quad a,b \in (1,2,3) \tag{14a}$$
$$m_{K^\pm}^2 f_K^2 = -\left[m_u^0\langle\bar{u}u\rangle_0 + m_s^0\langle\bar{s}s\rangle_0\right]; \quad a,b \in (4,5) \tag{14b}$$

$$m_{K^0}^2 f_K^2 = -\left[m_d^0 \langle \bar{d}d \rangle_0 + m_s^0 \langle \bar{s}s \rangle_0\right]; \quad a, b \in (6, 7) \tag{14c}$$

$$m_{88}^2 f_8^2 = -\frac{1}{3}\left[m_u^0 \langle \bar{u}u \rangle_0 + m_d^0 \langle \bar{d}d \rangle_0 + 4m_s^0 \langle \bar{s}s \rangle_0\right]; \quad a, b \in (8) \tag{14d}$$

$$m_{00}^2 f_0^2 = -\frac{2}{3}\left[m_u^0 \langle \bar{u}u \rangle_0 + m_d^0 \langle \bar{d}d \rangle_0 + m_s^0 \langle \bar{s}s \rangle_0\right]; \quad a, b \in (0) . \tag{14e}$$

and the one important off-diagonal mass matrix element is

$$m_{08}^2 f_0 f_8 = -\frac{\sqrt{2}}{3}\left[m_u^0 \langle \bar{u}u \rangle_0 + m_d^0 \langle \bar{d}d \rangle_0 - 2m_s^0 \langle \bar{s}s \rangle_0\right] . \tag{15}$$

We must be sure to distinguish between the "bare" f_a^p and dressed f_a ps decay constants. Similarly in the singlet channel

$$\mathbf{D}^{(0)}(q^2) = \frac{1}{\left(q^2 F(q^2) - m_{(0)}^2 (1 + 8G_A^{(0)}(f_0^p)^2 F(q^2))\right)}$$
$$\times \begin{pmatrix} g_p^2 \left(1 + 8G_A^{(0)} f_p^2 F(q^2)\right) & 4G_A^{(0)} m\sqrt{q^2} F(q^2) \\ 4G_A^{(0)} m\sqrt{q^2} F(q^2) & 2G_A^{(0)} q^2 F(q^2) \end{pmatrix} , \tag{16}$$

where

$$m_{(0)}^2 = (1/g_A^{(0)})m_{tH}^2 + m_{00}^2,$$

where, to leading order in N_C

$$m_{tH}^2 = \left(\frac{3g_{\pi qq}^2 G_{tH}}{4G_{eff}^2}\right) + \mathcal{O}(1/N_C^2) . \tag{17}$$

At the same time, the scalar mesons are not subject to mixing with other species, so their spectrum remains unchanged from that in Ref. [1].

Solutions to the BS Eq. in the vector and axial-vector channels As shown in Ref. [12], there is one solution to the BS Eq. (8) on the physical sheet, in the vector channel for sufficiently strong $G_V^{(1)}$ which smoothly connects to the one on the "second" sheet for arbitrarily weak $G_V^{(1)}$, see Fig. 3. A simple approximation to the flavour octet vector bound state mass

$$m_{V^{(1)}}^2 = \frac{3g_{A,Q}^{(1)} g_\pi^2}{4G_V^{(1)}} = 6m^2 \left(\frac{g_{A,Q}^{(1)}}{1 - g_{A,Q}^{(1)}}\right) . \tag{18}$$

is a good only as $g_{A,Q}^{(1)} \to 0$, or as $8f_\pi^2 G_V^{(1)} \to 1$, i.e., in the tight-binding limit, but otherwise overestimates the vector bound state mass, as can be seen in Fig. 3, where we show the numerical solutions to the vector channel BS Eq. (8) on the physical sheet of the S-matrix for fixed f_π. There we also show the Takizawa-Kubodera-Myhrer (TKM) "virtual bound state" mass below the critical coupling.

Similarly in the flavour singlet channel the vector bound state mass is

$$m_{V^{(0)}}^2 = \frac{3g_{A,Q}^{(1)} g_\pi^2}{4G_V^{(0)}} = \frac{3g_{A,Q}^{(0)} g_{\eta_0}^2}{4G_V^{(0)}} \tag{19}$$

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In the axial-vector sector

$$m^2_{A^{(8)}} = m^2_{V^{(8)}} + 6m^2 = \frac{6m^2}{1 - g^{(1)}_A} \tag{20a}$$

$$m^2_{A^{(0)}} = \frac{6m^2}{1 - g^{(0)}_A} \ . \tag{20b}$$

Note that the inequality $6m^2 \neq m^2_{A^{(0)}} - m^2_{V^{(0)}}$ is due to the, at least in principle, different values of the vector $G^{(0)}_V$ and axial-vector coupling constants $G^{(0)}_A$ in the flavour-singlet channel. This means that the value of $g^{(0)}_A$ cannot be expressed in terms of $m_{A^{(0)}}, m_{V^{(0)}}$, but one needs the constituent quark mass m instead of the vector meson one $m_{V^{(0)}}$, which, in turn, can be expressed in terms of $m_{A^{(8)}}, m_{V^{(8)}}$.

D. Mass relations

Scalar and pseudoscalar meson mass relations   Assuming $f^p_a = f^p$ for all $a = 0, 1, ...8$, [for corrections due to relaxation of this assumption, see Eq. (30b) in Ref. [13]] we find the following mass relations

$$m^2_{f_0} + m^2_{f'_0} - m^2_{K^{*+}_0} - m^2_{K^{*0}_0} = -m^2_{\text{tH}} \tag{21a}$$

$$m^2_{\eta} + m^2_{\eta'} - m^2_{K^+} - m^2_{K^0} = (1/g^{(0)}_A)m^2_{\text{tH}} \ , \tag{21b}$$

Eqs. (21a,e) lead immediately to the *modified* $U_A(1)$ sum rule

$$g^{(0)}_A \left[m^2_{\eta} + m^2_{\eta'} - m^2_{K^+} - m^2_{K^0} \right] \simeq m^2_{K^{*+}_0} + m^2_{K^{*0}_0} - m^2_{f_0} - m^2_{f'_0} + \mathcal{O}(1/N_C) \ , \tag{22}$$

where $\mathcal{O}(1/N_C)$ stands to remind us that the ratio of the two sides of the sum rule is subject to corrections of the order of 30% due to higher-order-in $1/N_C$ effects that have not been evaluated here [9]. This is the main result of this paper. It shows that one of the primary effects of the $U_A(1)$ symmetry-breaking 't Hooft interaction in the extended three-flavor NJL model with vector and axial-vector interactions is to produce a mass-squared splitting between the octet and the singlet for the scalar mesons that is smaller in size and opposite in sign to that of the pseudoscalar mesons. This brings the scalar meson effects of the 't Hooft interaction closer to those of the Veneziano-Witten one [13] [the latter predicts *no mass splitting* between the flavour-singlet and octet scalar mesons].

Vector and axial-vector meson mass relations   Following Eqs. (20a,b), we may write $g^{(0)}_A, g^{(1)}_A$ in terms of the meson masses in the tight-binding limit as follows

$$g^{(1)}_A = \frac{m^2_{V^{(8)}}}{m^2_{A^{(8)}}} \tag{23a}$$

$$g^{(0)}_A = 1 - \frac{m^2_{A^{(8)}} - m^2_{V^{(8)}}}{m^2_{A^{(0)}}} \ . \tag{23b}$$

The same results hold in the linear σ model with nucleon and vector and axial-vector meson d.o.f., see Ref. [14], only in that case the relations between $g_{A,N}$ and m^2_V, m^2_A are generally

true in the Born approximation there, rather than merely in the tight-binding approximation $g_A \to 0$. Hence the new sum rule Eq. (22) can be written as

$$
\left(m_{V^{(8)}}^2 + m_{A^{(0)}}^2 - m_{A^{(8)}}^2 \right) \left[m_\eta^2 + m_{\eta'}^2 - m_{K^+}^2 - m_{K^0}^2 \right] \simeq
$$
$$
m_{A^{(0)}}^2 \left[m_{K_0^{*+}}^2 + m_{K_0^{*0}}^2 - m_{f_0}^2 - m_{f_0'}^2 \right] + \mathcal{O}(1/N_C). \tag{24}
$$

III. COMPARISON WITH EXPERIMENT

In order to proceed we must determine the vector and axial vector octet and singlet masses $m_{V^{(8)}}, m_{V^{(0)}}, m_{A^{(8)}}, m_{A^{(0)}}$. Whereas the vector-meson octet is well established, the axial vector ones are not. For example, in the PDG 2000 tables (p. 118 in Ref. [3]) two axial vector "kaons" K_1^* are listed, one at 1270 MeV, another at 1400 MeV. The members of the $J^{PC} = 1^{++}$ and $J^{PC} = 1^{+-}$ octets are denoted as K_{1A}^* and K_{1B}^*, respectively, and described as "nearly equal mixes (45°) of the $K_1^*(1270)$ and $K_1^*(1400)$ ". This gives them a mass of 1337 MeV, which we shall use in subsequent calculations. Now we can check if the corresponding Gell-Mann–Okubo relations "work", specifically if

$$
\left(m_{f_1}^2 + m_{f_1'}^2 - 2m_{K_1^*}^2 \right) = 0? \tag{25}
$$

A deviation from zero of this quantity would move the mixing angle away from the "ideal" one (35.3^o). We find, using $m_{f_1}(1285)$ and $m_{f_1'}(1420)$ from Ref. [3],

$$
\Delta m_A^2 = \left(m_{f_1}^2 + m_{f_1'}^2 - 2m_{K_1^*}^2 \right) = (0.3 \text{ GeV})^2 , \tag{26}
$$

a small shift as compared with the absolute values of the masses involved. This fact modifies the singlet mass in the Gell-Mann–Okubo relations, which are given by

$$
m_{A^{(8)}}^2 = \frac{1}{3} \left(4m_{K_1^*}^2 - m_{A_1}^2 \right) = (1.36 \text{ GeV})^2 \tag{27a}
$$

$$
m_{A^{(0)}}^2 = \frac{1}{3} \left(2m_{K_1^*}^2 + m_{A_1}^2 \right) + \Delta m_A^2 = (1.35 \text{ GeV})^2 . \tag{27b}
$$

These essentially identical singlet- and octet axial-vector masses indicate that the two corresponding ENJL couplings, $G_A^{(0)}$ and $G_V^{(1)}$ are also very close to each other. Similarly for vector mesons

$$
m_{V^{(8)}}^2 = \frac{1}{3} \left(4m_{K^*}^2 - m_\rho^2 \right) = (0.93 \text{ GeV})^2 \tag{28a}
$$

$$
m_{V^{(0)}}^2 = \frac{1}{3} \left(2m_{K^*}^2 + m_\rho^2 \right) + \Delta m_V^2 = (0.89 \text{ GeV})^2 \tag{28b}
$$

$$
\Delta m_V^2 = \left(m_\omega^2 + m_\phi^2 - 2m_{K^*}^2 \right) = (0.25 \text{ GeV})^2 . \tag{28c}
$$

The proximity of the singlet- and octet vector meson masses indicates that another couple of (corresponding) ENJL couplings, $G_V^{(0)}$ and $G_V^{(1)}$ are essentially identical. Hence we may set $G_A^{(0)} = G_V^{(0)}$.

These results, in turn, determine the axial couplings of the constituent quarks. Note that Eqs. (23a,b) are not to be used for this purpose in general, as they are only accurate in the tight-binding limit. [Rather, their solutions can be thought of as upper limits on the allowed vector meson masses.] One must use Eqs. (6),(7) instead, which relate $g_A^{(1)}, g_A^{(0)}$ to the respective ENJL coupling constants $G_V^{(1)}, G_A^{(0)} = G_V^{(0)}$. The latter, in turn, are related to the vector meson masses via the (numerical) solution of the BS equation, which is shown in Fig. 3. There we can read off the value of $g_{A,Q}^{(0)} = g_{A,Q}^{(1)} \simeq 0.5$. Note that this is substantially lower than the "desirable" value of 0.75. This has serious consequences in the following.

A. The nucleon "spin problem"

It has long been known that the polarized deep-inelastic scattering spin asymmetry can be related to the nucleon's elastic axial current matrix element [8]. A consistent calculation of the nucleon axial current matrix element in the ENJL model ought to treat the nucleon on the same footing as the mesons, i.e., the nucleon ought to be the ground state solution to the 3-body Bethe-Salpeter equation. The nucleon as a relativistic three-constituent quark bound state is only starting to be explored within the minimal version of this effective model, i.e., without the vector and axial-vector interactions [15]. For this reason we shall not attempt a complete evaluation of $g_{A,N}^{(1)}$ in this model, but shall content ourselves with a simple estimate instead, accompanied with a commentary as to the shortcomings of the approach.

One prescription for estimating the isovector $g_A^{(3)}$ is based on the $SU(6)$ symmetric nucleon wave function and the impulse approximation result for the nucleon axial coupling in the quark model

$$g_{A,N}^{(3)} = \frac{5}{3}g_{A,Q}^{(1)} + g_{A,2Q}^{(1)}, \quad (1.2573 \pm 0.0028)_{\text{expt.}} . \tag{29}$$

Neglecting the two-quark contributions $g_{A,2Q}^{(1)}$, the ENJL model predicts $g_{A,N}^{(3)} = 5/6 = 0.83$. Such "negligence" is known to be in conflict with the chiral Ward idenities of the theory [16], however. On general grounds we can only say that $g_{A,2Q}^{(3)} \simeq \mathcal{O}(1/N_C)$, where $N_C = 3$ [9]. Hence, the "experimental" value $g_{A,Q}^{(1)} = 0.754$ is consistent with the ENJL result $g_{A,Q}^{(1)} = 0.5 \pm 0.33$.

Evaluation of the two-quark contribution in the ENJL model is technically demanding and will have to await the solution of the three-quark Bethe-Salpeter equation [15] and the construction of the corresponding two-body axial currents. It ought to be pointed out, however, that Morpurgo [17] has extracted baryon matrix elements of some two-quark currents from experimental data and that his findings are in accord with the above order-of-magnitude estimate [1].

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[1]Morpurgo's treatment of the two-quark contributions is a phenomenological one, however: he does not assume a specific two- or three-quark potential/dynamics and then derive (via PCAC) the corresponding two- and three-quark axial currents. Rather, he makes a general parametrization of such operators and their nucleon matrix elements and then determines the latters' values from the experiment. Note that this procedure is different from what is proposed here for future work.

Completely analogous results hold in the flavour-singlet channel

$$g_{A,N}^{(0)} = g_{A,Q}^{(0)} + g_{A,2Q}^{(0)}, \quad (0.12 \pm 0.23)_{\text{expt.}} \ . \tag{30}$$

and in the eighth member of the octet channel

$$g_{A,N}^{(8)} = (3F - D)g_{A,Q}^{(1)} + g_{A,2Q}^{(8)}, \quad (0.579 \pm 0.025)_{\text{expt.}} \ . \tag{31}$$

where the SU(6) factor $3F - D = 1$ (instead of $5/3 = 1 + F/D$ in the isotriplet channel). These results are also subject to (different) axial MEC corrections, which are also not known at the moment, except that they are of $\mathcal{O}(1/N_C) \simeq 33\%$, and that they *must* be included if chiral symmetry is to be conserved [16]. It seems fair to say that these two ENJL predictions, $g_{A,N}^{(0)} = g_{A,N}^{(8)} = 0.5 \pm 0.33$, are also within the (quite large) theoretical uncertainty (± 0.33) from the experimental values $g_{A,N}^{(0)} = 0.12 \pm 0.23$ and $g_{A,Q}^{(8)} = 0.579 \pm 0.025$.

One's best hope for a "peaceful" resolution of this conflict in the ENJL model is that the two-quark axial currents are large and satisfy the following two inequalities, $g_{A,2Q}^{(0)} \leq 0$, and $g_{A,2Q}^{(1)} \geq 0$, which would bring the observed axial couplings into better agreement with the ENJL predictions. There is stil some "wiggling space" left in the form of unknown quark masses and perhaps less in the form of imperfectly known axial vector spectra, but it is small in both instances.

The relations between proton spin fractions Δq carried by specific quark flavour q and the axial couplings are as follows

$$
\begin{aligned}
g_A^{(0)} &= \Delta u + \Delta d + \Delta s \\
g_A^{(3)} &= \Delta u - \Delta d \\
g_A^{(8)} &= \Delta u + \Delta d - 2\Delta s \ .
\end{aligned}
\tag{32}
$$

Using the one-body predictions of the axial couplings in the ENJL model one is led to the following numerical results $\Delta u = 0.67 \pm 0.33$, $\Delta d = -0.17 \pm 0.33$ and $\Delta s = 0 \pm 0.33$, all subject to the large $\mathcal{O}(1/N_C)$ theoretical uncertainties. These results can be compared with the (old) EMC data [2] $\Delta u = 0.78 \pm 0.08$, $\Delta d = -0.50 \pm 0.08$ and $\Delta s = -0.16 \pm 0.08$, or with the more recent numbers [19] $\Delta u = 0.830 \pm 0.077$, $\Delta d = -0.437 \pm 0.073$ and $\Delta s = -0.093 \pm 0.043$. We see that the ENJL predictions are, given the large theoretical uncertainties, in better agreement with experiment than the axial couplings themselves.

The vanishing axial coupling/spin content carried by the strange quarks is not an intrinsic deficiency of this model, which allows for polarized $s\bar{s}$ sea, but rather a consequence of the (almost) ideally mixed vector and axial-vector meson spectra, which were taken from experiment and led to $g_{A,Q}^{(0)} \simeq g_{A,Q}^{(1)}$ and hence to $\Delta s = 0$. The trouble is that the "nucleon spin" experimental results Eqs. (29),(30) indicate that this should not be the case. The two-quark axial current contributions may yet change that conclusion.

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[2] Spin asymmetries are not commonly extracted from the DIS experiments any more, since it has become clear that there are other (orbital and gluon) contributions that exist but cannot be experimentally separated [18].

The large theoretical uncertainties allow only weak conclusions to be drawn at this moment: The "nucleon spin measurements" and the vector, axial-vector meson spectra are inconsistent with each other in the ENJL model, but theoretical uncertainties in the form of two-quark axial currents are potentially large. Even when these two-quark axial currents are calculated we will have no reason to be overly optimistic about their future predictions of deep-inelastic sum rules, as this model does not include gluons [3]. Inclusion of gluons into this model would make it (almost) as intractable as QCD, however.

B. Scalar-pseudoscalar-vector-axialvector meson mass sum rule

Using Eqs. (27a,b) and (28a,b), the sum rule Eq. (24) can be rewritten as

$$\left[4m_{K^*}^2 - m_\rho^2 + 3\left(m_{f_1}^2 + m_{f_1'}^2\right) + 2\left(m_{A_1}^2 - 4m_{K_1^*}^2\right)\right]\left[m_\eta^2 + m_{\eta'}^2 - m_{K^+}^2 - m_{K^0}^2\right] \simeq$$
$$\left[3\left(m_{f_1}^2 + m_{f_1'}^2\right) + \left(m_{A_1}^2 - 4m_{K_1^*}^2\right)\right]\left[m_{K_0^{*+}}^2 + m_{K_0^{*0}}^2 - m_{f_0}^2 - m_{f_0'}^2\right] + \mathcal{O}(1/N_C). \qquad (33)$$

This sum rule can be used in the same way as the one in Ref. [1]: it predicts the "second" isoscalar scalar meson f_0' given the masses of the "first" isoscalar scalar f_0, and the scalar "kaon" K_0^*, as we already know the masses of all the pseudoscalar mesons fixing the left-hand side (lhs) of the sum rule as $m_{tH}^2 = 0.72 \text{ GeV}^2 = (855 \text{ MeV})^2$, as well as all the vector and axial-vector meson masses. The scalar "kaon" is $K_0^*(1430)$; choosing $f_0(1500)$ as one member of the f_0, f_0' pair, the sum rule predicts $m_{f_0'} = 1.21$ GeV. The closest observed state is $f_0(1370)$. Choosing $f_0(1370)$, the sum rule predicts $m_{f_0'} = 1.36$ GeV, the closest observed state being $f_0(1500)$. And finally, picking $f_0(1710)$ one finds $m_{f_0'} = 0.89$ GeV, the closest observed state being $f_0(980)$. We leave it to the interested reader to pick his/her favourite pair.

The problem of scalar mesons is manifestly not solved by this sum rule, rather the problem merits further study. Indeed, there are many other models of scalar mesons "on the market" today, some with substantialy different physical pictures, see e.g. the papers cited in Ref. [20]. We have not tried to explore here the relation of this model to others, so as not to stray too far afield. Rather, we hope to have convinced the reader that a new aspect - the vector and axial-vector meson masses, or constituent quark axial-vector couplings - unexpectedly enters the subject.

IV. SUMMARY AND CONCLUSIONS

In conclusion, we have shown that the $U_A(1)$ symmetry-breaking 't Hooft interaction causes a mass splitting between the singlet and octet scalar mesons in this effective chiral field theory. This scalar meson mass splitting is related to the analogous pseudoscalar

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[3]Admittedly, some of the gluon contributions (those of the $U_A(1)$ anomaly) are accounted for by the 't Hooft interaction in this model, but that still leaves one without gluonic partons.

meson mass splitting and the constituent quark flavour-singlet axial coupling constant that, in turn, can be related to the so-called nucleon spin problem. The latter two phenomena are well known manifestations of $U_A(1)$ symmetry-breaking [8]. In this model, however, the constituent quark flavour-singlet and octet axial coupling constants are almost entirely determined by the vector and axial-vector meson spectra and very little by the 't Hooft interaction. As these spectra are almost ideally mixed there is virtually no flavour-mixing "strangeness-induced" axial coupling and hence no polarized $s\bar{s}$ sea quarks. [4]

These results are in disagreement with experiment, albeit subject to some potentially large theoretical corrections, so it is not quite clear yet if one ought to declare the model dead and abondon it, or work harder to improve it. On the one hand, one ought not be surprised by the large discrepancy, as the model is simple and cannot pretend to be the whole story (there is no confinement). On the other hand, it manifestly shows that the correct implementation of chiral symmetry is insufficient to reproduce the observed nucleon axial couplings.

We have thus established a new connection between two apparently unrelated kinds of physics (deep inelastic scattering and light meson spectra) and hope thus to have opened a discussion of this potentially significant question in other models and in QCD. We expect that similar effects ought to exist in the Coulomb gauge QCD models of mesons, such as those in Refs. [2,21], but have not been discussed explicitly thus far.

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[4] The phenomenon of axial coupling constant renormalization occurs in effective field theories of the linear σ model type with fermion and vector and axial-vector meson d.o.f., see Ref. [14], only there the linear relations between $g_{A,N}$ and m_V^2, m_A^2 are always true in the Born approximation, rather than being true merely in the tight-binding approximation, as is the case here. Indeed one may argue that the aforementioned model of Ref. [14] can be obtained as the result of the bosonization of our effective chiral fermion field theory.

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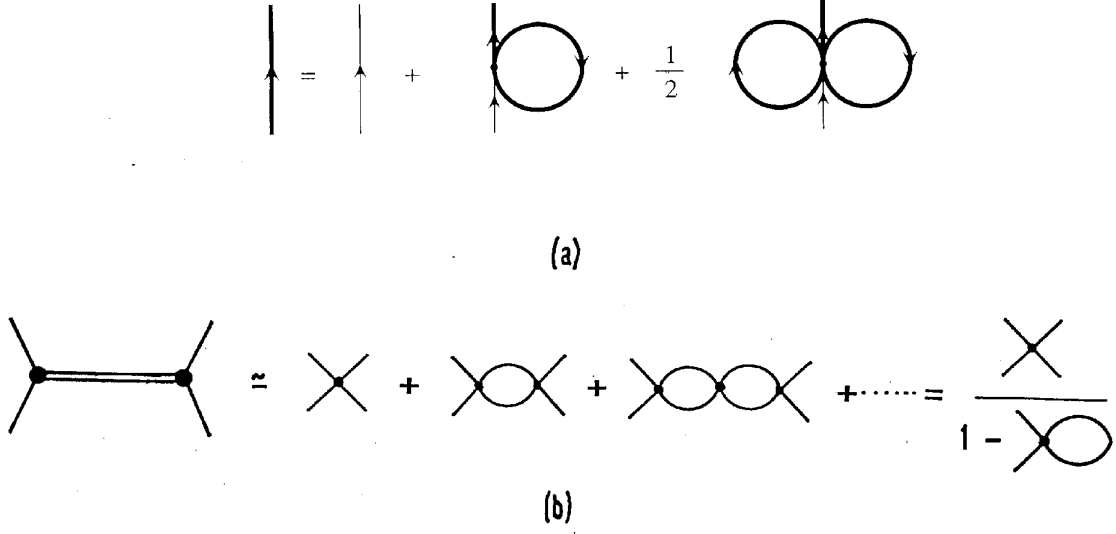
(a)

(b)

FIG. 1.   The two Schwinger-Dyson equations determining the dynamics of this model in the leading-order in $1/N_C$ [Hartree + RPA] approximation: (a) the one-body, or "gap" equation; and (b) the two-body, or Bethe-Salpeter equation. The thin solid line is the bare, or "current" quark, the heavy solid line is the constituent quark and the double solid line is a composite (bound state) scalar, pseudoscalar, vector-, or axial-vector boson.
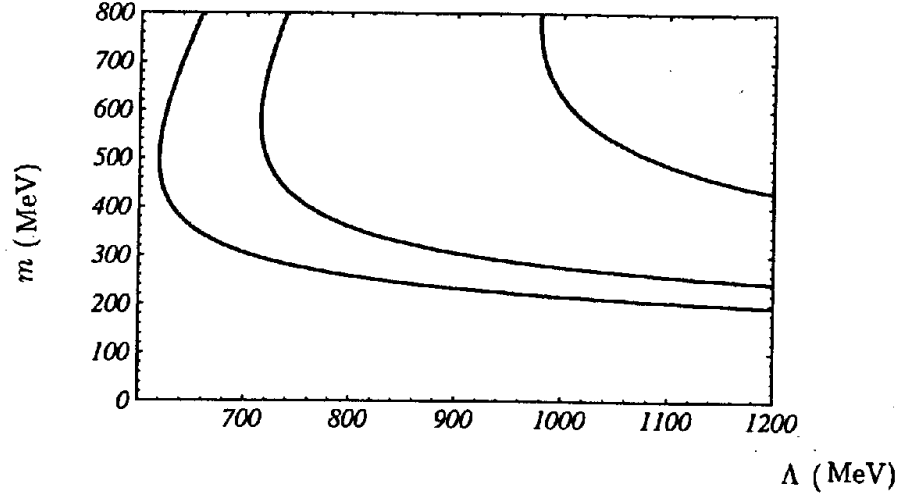
FIG. 2.   The constituent quark mass m as a function of the Pauli-Villars [PV] cutoff Λ (in units of MeV) $g_A = 1, 0.75, 0.4$, (the far left h.s., the middle and the far right h.s. curves, respectively) at fixed $f_\pi = 93$ MeV.
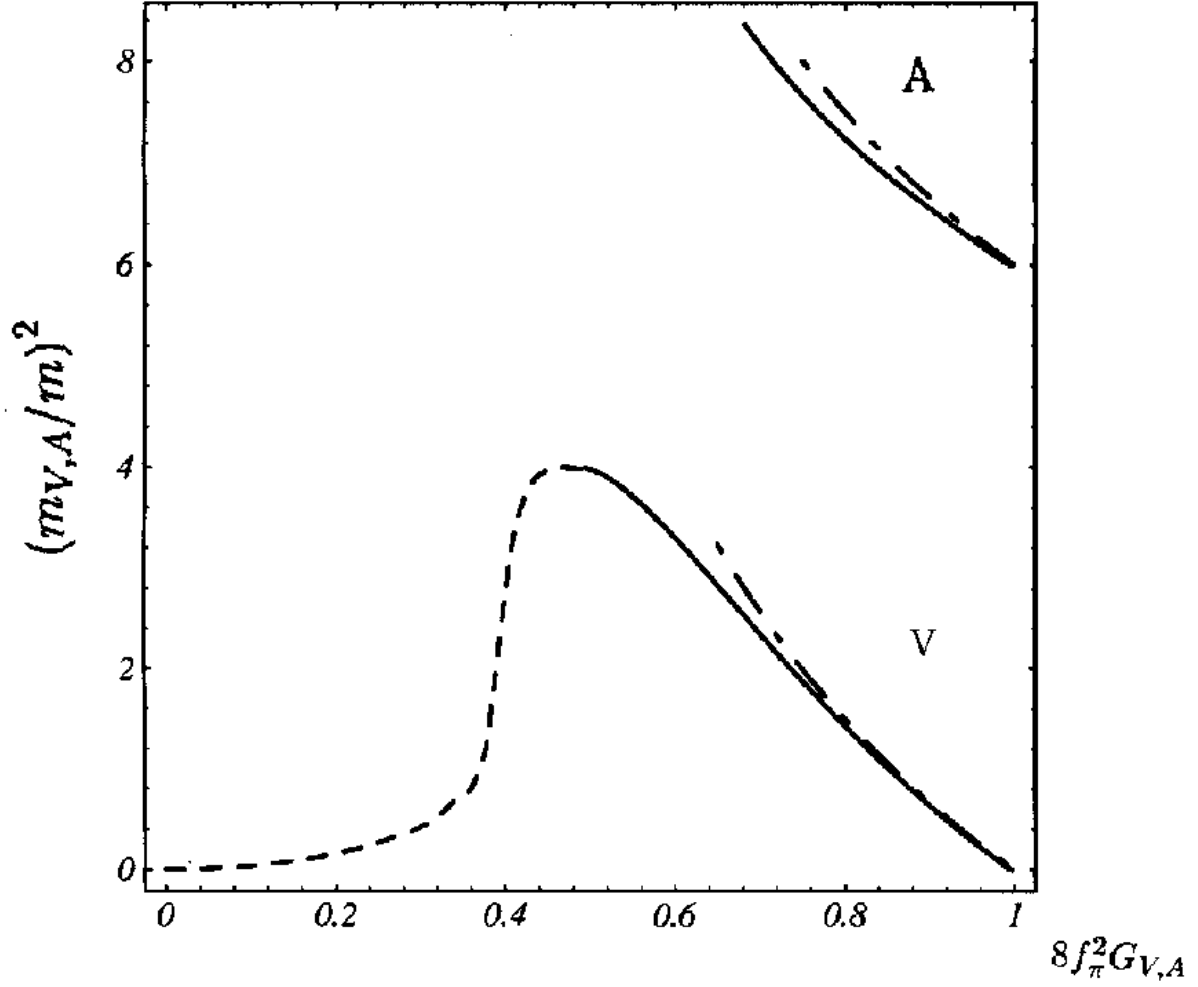
FIG. 3. Solutions to the BS equation [vector-, or axial-vector state mass squared $m_{V,A}^2$ rescaled by the constituent quark mass squared m^2] as functions of the rescaled vector interaction coupling constant $8f_\pi^2 G_{V,A}$, with $m = 313$ MeV in the ENJL model. [The continuum threshold is at 4.] (1) vector bound state with fixed $f_\pi = 93$ MeV [lower solid line denoted by V] continuing into the Takizawa-Kubodera-Myhrer [TKM] "virtual bound state" with fixed f_π [short dashes] at lower values of G_V; (2) root of the real-part of the axial-vector BS Eq. with fixed f_π [solid line denoted by A]; (3) analytic approximations to the vector bound state mass Eqs. (18), (19) and the axial-vector mass Eq. (20a,b) at fixed f_π [dot-dashes].